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ADDENDUM TO 'A STUDY ON THE EIGEN-MODES OF PCF VARYING THE POSITION OF THE DIELECTRIC HOLES BY FEM'

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ABSTRACT

This addendum to a previous studies show the 3-dim. characteristics of the eigen-modes of PCF. It describe about the spatial distributions of the eigen-modes in the central region of PCF surrounded by the dielectric holes. The spectra for the transverse eigen-modes of PCF are similar to the previous studies. In addition, these spectra reveal especially the characteristic of the electromagnetic potential along the fiber direction. FEM has been constructed basing on the 3-dim. vector Helmholtz equation. The matrix eigen-equations have been solved with the Krylov-Schur iteration algorism. As results, the eigen-modes of the transverse field and electromagnetic potential are illustrated together visually through the spectra schematically.

Keywords: *FEM, PCF, eigen-mode, dielectric, waveguide*.

I. INTRODUCTION

Previously, FEM(Finite Element Method) has been carried out to investigate the eigen-modes of PCF(Photonic Crystal Fiber) [1]. PCF model was constructed with background material of high-index dielectric and the several circular holes of low-index one. PCF was the square form and the central region of it was surrounded by the dielectric holes. There are various ways for identifying the property of the eigen-mode of PCF. The previous study has focused on the spectra resulting from the cross section of PCF. As a final result, the spectra of TE(Transverse Electric) and TM(Transverse magnetic) modes have been illustrated in the schematic representation. From the spectra, it has been identified that the dielectric holes restrict the electromagnetic field into high-index dielectric region. Basing on this property, it was confirmed that the electromagnetic field could be confined in the specific region by arranging the dielectric holes systematically.

But the eigen-modes have been accompanied with various limitations in understanding the eigen-property, because the spectra was represented by two-dimensional feature. The property of the electromagnetic potential was excluded so that the field strength along this direction could not understand directly from the spectra. The electromagnetic potential profile of the eigen-mode might give the more valuable information about PCF propagating property. It would be needed that the eigen-mode involving the transverse and the electromagnetic potential is calculated simultaneously. So, in this addendum, the 3-dim. Helmholtz equation was calculated to obtain the more precise characteristics of the eigen-mode of PCF.

The eigen-equation was constructed from FEM based on the vector 3-dim. Helmholtz equation. The eigen-equation was divided into two equations for the transverse field and electromagnetic potential components. FEM has been carried out using the Galerkin method of weighted residuals to construct the linear equations. These equations were represented by the matrix eigen-equations and assembled into a global matrix eigen-equation. The eigen-modes were calculated using Krylov-Schur iteration method [2]. The eigen-modes of the transverse field and electromagnetic potential were simultaneously obtained from the similarity transformation matrix which make the global eigen-matrix into a Schur form. As a final result, the spectra of the eigen-modes have been illustrated in schematic representations for the transverse field and electromagnetic potential components simultaneously.

II. FEM FORMULATION

The characteristic eigen-mode of PCF depend on the spacial distribution of the dielectric holes. As in the previous study, PCF was assumed to be constructed with background material of high-index dielectric and the several circular holes of low-index one. Fig.1 show the cross section of PCF whose space was divided into triangular mesh. The radius of the dielectric holes were sufficiently large so that they overlap with each other at the central region of PCF.

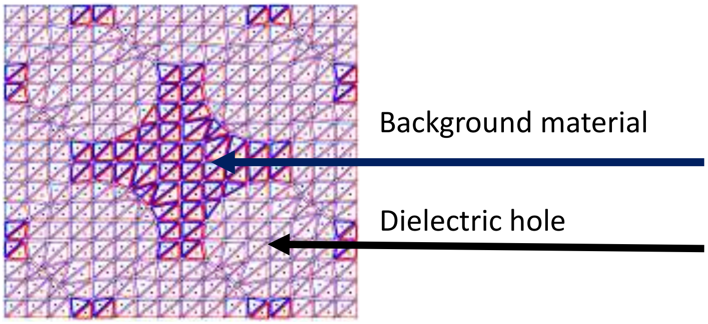

Figure 1. The cross section of PCF and the mesh

The vector Helmholtz equation was used to obtaining the eigen-mode of PCF [3][4]. It is described as following equation

$$\vec{\nabla} \times \left(p\vec{\nabla} \times \vec{F}\right) - k_o^2 q\vec{F} = 0 \quad (1)$$

where k_o is the wavenumber and for $\vec{F} = \vec{E}$(*electric field strength*), $p = 1/\mu_r(\mu_r\!: relative\ permeability\ \mu/\mu_o)$, $q = \varepsilon_r(\varepsilon_r\!: relative\ permitivity\ \varepsilon/\varepsilon_r)$, and for $\vec{F} = \vec{H}$(*magnetic field strength*), $p = 1/\varepsilon_r$, $q = \mu_r$. For convenience of the calculation, $\vec{F}, p\ and\ q$ was used to relate the physical values without differentiating the electric and magnetic field modes. Basing on this equation, the eigen-equation was constructed from the FEM which used the Galerkin method of weighted residuals. The shape function was calculated for the triangular element mesh using its node points

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \quad (2)$$

where $a_i = x_j y_k - x_k y_j, b_i = y_j - y_k\ and\ c_i = x_k - x_j (i, j\ and\ k\ are\ cyclical\ ordering)$ and A is the area of the element triangular mesh. Relating with these shape functions, the tangential edge vectors for each element mesh are given by

$$\overrightarrow{W_m} = L_m\left(N_i \vec{\nabla}_t N_j - N_j \vec{\nabla}_t N_i\right) \quad (3)$$

where $\vec{\nabla}_t = \frac{\partial}{\partial x}\hat{x} - \frac{\partial}{\partial y}\hat{y}$, L_m is length of the edge connecting nodes $i\ and\ j$. It is more convenient to express these vectors by using the shape function coordinates.

$$\overrightarrow{W_m} = \frac{L_m}{4A^2}\left[(A_m + B_m)\hat{x} + (C_m + D_m)\hat{y}\right] \quad (4)$$

where $A_m = a_i b_j - a_j b_i$

$B_m = c_i b_j - c_j b_i$

$C_m = a_i c_j - a_j c_i$

$D_m = b_i c_j - b_j c_i = -B_m$

With these tangential edge vectors and shape functions, the transverse vector fields and their electromagnetic potential component of each element mesh can be written as

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = \begin{bmatrix} \{W_x\}^T & 0 \\ \{W_y\}^T & 0 \\ 0 & jN^T \end{bmatrix} \begin{Bmatrix} F_t^e \\ F_z^e \end{Bmatrix} \quad (5)$$

The envelope function concept for the element vector

$$\{F\} = \begin{Bmatrix} \vec{F}_t \\ F_z' \end{Bmatrix} \exp(-j\beta) \quad (6)$$

is used where β is a propagation constant. The eigen-equation was obtained from the (0,1) Pade approximation of propagation scheme at the beginning position [5]

$$[A]\{F\} = -\frac{1}{2k_o n\beta}[B]\{F\} \quad (7)$$

Where $[A] = \begin{bmatrix} [G] & [E] \\ [F] & [D] - k_o^2[I] \end{bmatrix}$

$[B] = \begin{bmatrix} k_o^2 n_o^2[G] - k_o^2[H] + [C] & k_o^2 n_o^2[E] \\ k_o^2 n_o^2[F] & k_o^2 n_o^2[D] - k_o^2[I] \end{bmatrix}$

and for the calculation stability $F_z' = j\frac{\partial}{\partial z}F_z$ was adopted. The matrices components are calculated as following

$[C] = \iint_A p(\vec{\nabla}_t \times \vec{W}_m) \cdot (\vec{\nabla}_t \times \vec{W}_n) ds = p\frac{L_m L_n}{4A^3} D_m D_n$

$[D] = \iint_A p(\vec{\nabla}_t N_i) \cdot (\vec{\nabla} N_j) ds = p\frac{b_i b_j + c_i j}{4A}$

$[E] = \iint_A p\vec{W}_m \cdot (\vec{\nabla} N_j) ds = p\frac{L_m}{8A^2}[b_j(A_m + B_m \bar{y}_{tri}) + c_j(C_m + D_m \bar{x}_{tri})]$

$[F] = \iint_A p(\vec{\nabla} N_i) \cdot \vec{W}_n ds = p\frac{L_n}{8A^2}[b_i(A_n + B_n \bar{y}_{tri}) + c_i(C_n + D_n \bar{x}_{tri})]$

$[G] = \iint_A p\vec{W}_m \cdot \vec{W}_n \, ds = p\frac{L_m L_n}{16A^3}\sum_{a=1}^{a=5} I_a$

$[H] = \iint_A q\vec{W}_m \cdot \vec{W}_n \, ds = q\frac{L_m L_n}{16A^3}\sum_{a=1}^{a=5} I_a$

where $I_1 = (A_m A_n + C_m C_n)$

$I_2 = (C_m D_n + D_m C_n)\bar{x}_{tri}$

$I_3 = (A_m B_n + B_m A_n)\bar{y}_{tri}$

$I_4 = \frac{B_m B_n}{12}(\sum_{i=1}^{i=3} y_i^2 - 9\bar{y}_{tri}^2)$

$I_5 = \frac{D_m D_n}{12}(\sum_{i=1}^{i=3} x_i^2 - 9\bar{x}_{tri}^2)$

and $[I] = \iint_A q N_i N_i ds = q\frac{A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

In these representations, subscripts for N and $\vec{W}$ indicate the node and edge numbers respectively. These element matrices have been assembled over all triangular elements to obtain a global eigen-matrix equation.

As mentioned in the previous study, it has been well known that the Krylov-Schur iteration method is the most reliable technique for finding the prominent eigen-modes [6]. The method would be more efficiently implemented in finding specific eigen-pairs by performing the shift-invert strategy as following [7]

$$\lambda_o\{F\} = \frac{[B]}{[A] - \sigma[B]}\{F\} = [M]\{F\} \quad (8)$$

where $\lambda_o = 1/\frac{\sigma}{2nk_o}$. The sparsity and symmetry of the eigen-equation would be lost, but by this strategy the convergent rate is more promoted at the specific value σ. Subsequently, the Krylov-Schur iteration method is performed on this square matrix $[M]$.

III. RESULT & DISCUSSION

In this study, FEM was carried out to investigate the eigen-mode of PCF involving dielectric holes. The attention of previous studies were on the change of transverse eigen-modes according to addition of the dielectric holes to PFC. From results, it was known that the eigen-modes were depend on the spatial distribution of the dielectric holes and their dielectric values. As gradually the central region surrounded by the dielectric holes, the electromagnetic field was restricted to this region without any leakage. For a better understanding to the eigen-mode, it may need to know the strength profile of the electromagnetic potential. So the previous study has been extended to 3-dim. problem to identify the field strength of eigen-modes along the longitudinal direction of PCF. FEM formulation is different from the previous studies. It contains another variable corresponding to the longitudinal direction of the PCF as described in eq. (5). The first two components represent the eigen-modes of the transverse electromagnetic field and the last one reflect the electromagnetic potential corresponding to them.

The Krylov-Schur iteration method has been applied to the square matrix $[M]$ of eq. (8). This iteration method similarly transformed the square matrix $[M]$ into the Schur square matrix. The similarly transformation matrix has the dimension $\mathrm{n} \times 20$ where n is consisted with the edge and node numbers of the triangular mesh and 20 is the dimension of the Arnodi contraction matrix. Each columns of the transformation matrix describe a specific eigen-mode generated in the waveguide. As in the previous study, the lateral surface of the waveguides were assumed to be perfect conductor. The reason for it was that this assumption is convenient to obtain TM modes by ignoring the variables on the surface. As a result, several prominent eigen-modes are revealed in fig.2 schematically.

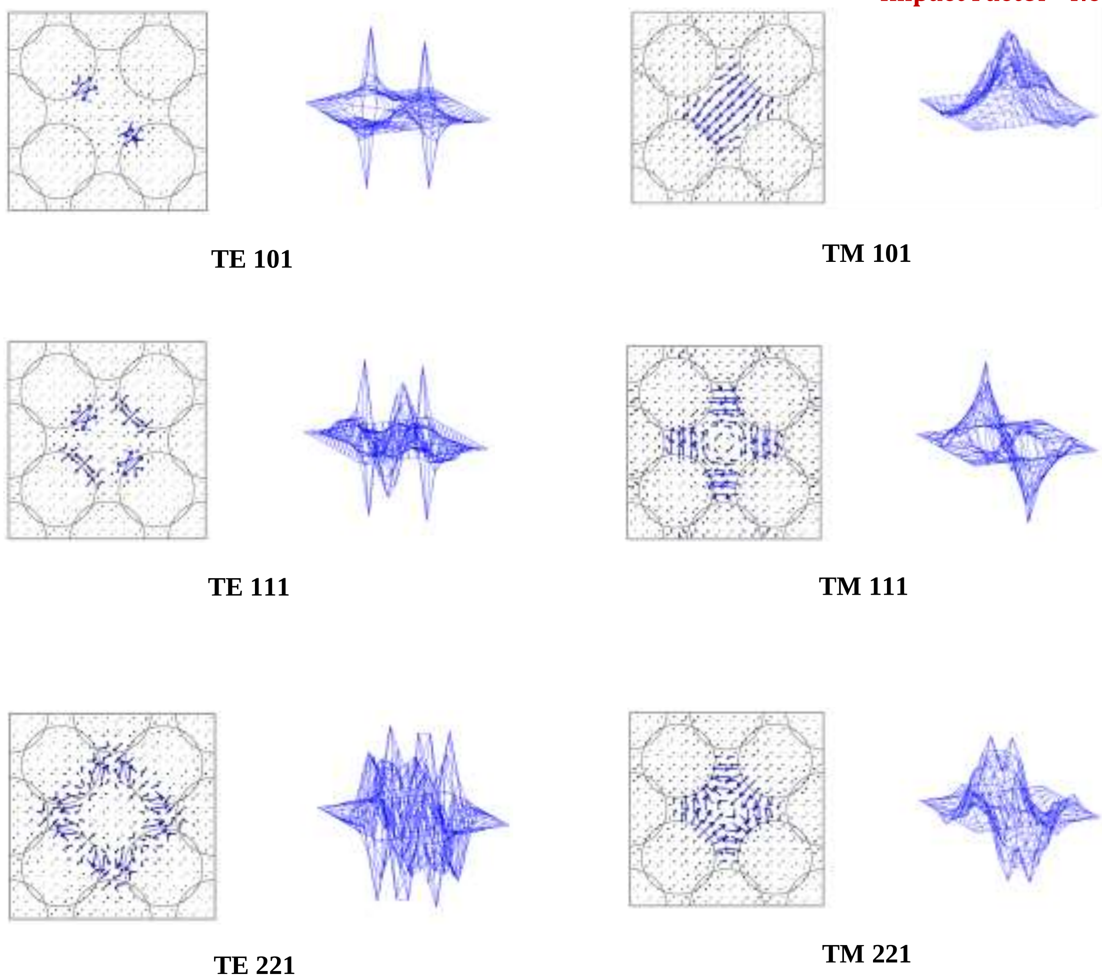

Figure 2. The eigen-modes of PCF

The spectra for each eigen-modes were composited with the transvers field and electromagnetic potential. In transvers eigen-mode representation, the stronger the brightness of the blue color indicates the more intensified electromagnetic field. As can be identified from these eigen-modes, the transverse electromagnetic field is confined in the central region surrounded by the dielectric holes. This trend is consistent with the spectra representing the spatial distribution of the electromagnetic potential. The eigen-modes representing the transverse field are accompanying with the electromagnetic potential eigen-modes as can be seen in the fig.2. These spectra show that the major peak is considerably gathered in the central region. The peak strength and sharpness of the electromagnetic potential indicated the magnitude of the transvers electromagnetic field. The space gradient of the electromagnetic potential is proportional to the electromagnetic field strength. One of the eigen-mode make possible to understand the characteristics of other. From these results, it could be identified that the eigen-modes of the transverse field together with them of the electromagnetic potential are consistently described the confining phenomenon of electromagnetic field in the central region of PCF.

IV. CONCLUSION

This addendum to the previous study show the 3-dim. characteristics of the eigen-modes of PCF. As in the previous studies, it could be identified that the dielectric holes restrict the electromagnetic field into the central region of PCF. The eigen-modes of the transverse field and those of the electromagnetic potential are mutually complementary to describe the distribution characteristic of electromagnetic field in PCF.

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